Stability of systems of general functional equations in the compact-open topology

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Abstract

We introduce a fairly general concept of functional equation for k-tuples of functions $f_1, \ldots, f_k\ X \to Y$ between arbitrary sets. The homomorphy equations for mappings between groups and other algebraic systems, as well as various types of functional equations and recursion formulas occurring in mathematical analysis or combinatorics, respectively, become special cases (of systems) of such equations. Assuming that X is a locally compact and Y is a completely regular topological space, we show that systems of such functional equations, with parameters satisfying rather a modest continuity condition, are stable in the following intuitive sense: Every k-tuple of "sufficiently continuous," "reasonably bounded" functions $X \to Y$ satisfying the given system with a "sufficient precision" on a "big enough" compact set is already "arbitrarily close" on an "arbitrarily big" compact set to a k-tuple of continuous functions solving the system. The result is derived as a consequence of certain intuitively appealing "almost-near" principle using the relation of infinitesimal nearness formulated in terms of nonstandard analysis.